

Models for Economic Most Advantageous Tenders

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Samenvatting

In de praktijk worden vele vergelijkingen gebruikt om EMVI-scores (Economisch Voordeligste Inschrijving, of - in iets onbehulpener Nederlands - Economisch Meest Voordelige Inschrijving) van aanbiedingen te bepalen. Naast juridische zijn ook numerieke aspecten al vaak ter discussie geweest. Desondanks blijken er nog steeds onduidelijkheden voor te komen. Daarom lijkt een wat fundamenteelere aanpak van tenminste het tweede aspect zinvol. Gelukkig is kennis hiervan ruim voorhanden in onder andere de traditionele micro-economie en keuze- en besliskunde.

In dit artikel wordt ingegaan op:

- enkele beginselen, begrippen en gereedschappen uit de economie en besliskunde
- een eenvoudig EMVI-model, dat kan worden opgevat als een linearisering van andere EMVI-modellen
- enkele ingewikkeldere modellen die voor gebruik in de praktijk zijn voorgesteld
- enkele praktische moeilijkheden bij EMVI-modellen

De voornaamste bevindingen in dit artikel zijn:

- de discussie kan beperkt worden door de economische beginselen voor ogen te houden
- eenvoudige modellen verdienen de voorkeur, omdat in de regel de gegevens die nodig zijn voor complexere modellen onvoldoende bekend zijn en aannamen onvoldoende onderbouwd kunnen worden
- aanbesteders moeten onvermijdelijk keuzen maken, waarvoor deskundigheid vereist is en waarvoor zij verantwoordelijk zijn en gesteld horen te worden.

Leeswijzer

Het artikel is geschreven naar aanleiding van een ééndaagse bijeenkomst georganiseerd door PIANOo over 'de wiskundige basis van EMVI beoordelingsmethodieken'. De formules, gebruikelijk in de wiskunde, zijn, hoewel zij op zich volstaan, toegelicht met figuren en tekst en een eenvoudig fictief voorbeeld. Omdat het betoog is dat bij de complexere modellen het verkrijgen van betrouwbare informatie te lastig is, zijn hier geen voorbeelden gegeven. Zij zouden als vergelijking met het gegeven voorbeeld weinig toevoegen.

Het artikel kan ook gelezen worden zonder de formules en figuren te beschouwen. In dat geval moet de lezer maar aannemen dat de slotopmerkingen ten dele door deze formules onderbouwd zijn.

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1 Introduction

In practice a large number of equations for calculating EMAT-scores (Economic Most Advantageous Tender) on offers have been proposed and used. The juridical and numerical aspects have been topics of much discussion and comparative study. These efforts however left unclarified matters and a somewhat more fundamental approach seems appropriate. Fortunately the basic knowledge of such problems is already long time known in micro-economical- and choice- and decision theory. In this paper, some of these basics are elaborated to formulate the probably simplest valuation- and choice model. Next some equations from practice are compared with regard to these principles and some effects are discussed ¹.

Some regularly seen secondary problems spring from unclear definitions and ambiguous, homonymic and synonymic terminology. Concepts and choices between alternative terms are discussed. Furthermore simple formulae and numbers – sometimes indicated as mathematics – are more univocal and suitable to avoid misunderstandings than text. To reduce the apparent abstractness of formulae, they are illustrated by graphs for the simplest case of two independent variables.

2 Micro-economics

2.1 Exchange and choice

In the heart of the matter micro-economics is about need and scarcity, resulting in choice and exchange. The two main forms are one participant choosing between multiple goods and multiple participants agreeing on one price for one good ². In the simplest form, choice concerns one participant and two options and price two participants and one option. The participants are characterised by their relevant interests and the options by their relevant attributes. The compound case of several participants, several options and multiple rounds implies in practical situations often negotiation (which term originally only indicates doing business, i.e. being busy). This case is beyond the scope of this paper as well as temporal and probability matters.

All of these forms require the possibility of quantification or at least ranking, so adding numbers to the options. These numbers reflect the value that one participant adds to an option. Valuation may therefore be regarded a limiting case of one participant and one option (take or leave). In general all four forms may occur in tenders.

table 1: overview of theories

| theories | 1 participant | m participants |
|----------|---------------|------------------|
| 1 option | value | exchange (price) |

¹ PIANOo, the Dutch Public Procurement Expertise Centre, part of the Ministry of Economic Affairs of The Netherlands, organised a one-day conference to discuss ‘the mathematical basics of EMAT judgement methods’ on 28 January 2015, to which this paper is partly related.

² As the term ‘good’ (commodity) is rather typical for the domain of economy, ‘option’ will be used for matters of choice. A good is usually a product or a service. An option is a thing that may be chosen. Instead of option, the possibly less correct term ‘alternative’ is more usual in decision theories. The other meanings of option - of right to choose and the commercial one of right to buy or sell - are not meant here.

| | | |
|-----------|-------------------|------------|
| n options | choice (decision) | vote, game |
|-----------|-------------------|------------|

The traditional way of contracting by governments is a simple choice on lowest bid-price. EMAT is a more complex choice, in which the bid-price is one of the aspects, usually next to quantity or quality³ aspects of the bid. Exchange and game mechanisms are dominant in business, while governments are more bound to balancing economic principles with societal and political values than enterprises⁴.

2.2 Value

Value is a concept with several meanings. The most general is a comparison with something else. Another meaning is value for an application i.e. usefulness or utility. A third meaning is a subjective an unspecified award by someone to something that is found valuable or precious. In Value Methodology, value is defined as performance (of a function) per cost (of acquiring that function). Worth is the value for an individual and therefore value is the average worth of a group of people [Stewart 2005]. Value is a magnitude and therefore expressed as a number. In this paper the first meaning is meant, though that does not exclude the second and third in any case. In the context of EMAT, value is often expressed as money and unit value as money per unit quantity. Money has at least three meanings, namely replacement medium of exchange, unit of account and saving. The first meaning is also historically the first: money serves to decrease unnecessarily dragging around and storage of goods. The second meaning is the direct consequence of the first. Savings give rise to time-effects like interest. Here the first meaning should be kept in mind (money as a kind of good).

Variables in an EMAT choice model have at least a name, specification, quantity and a dimension. The purchaser will have to determine them. So numbers are indispensable and, if a measure of comparability is available, one can measure or estimate the independent variables:

- measuring assumes a standard for reference; a difference between the known reference and the unknown variable can be determined
- an estimation can be made if there is a non-standard reference; a ratio between the known and the unknown variable can be determined

There are many techniques available to determine a reference, even in case of subjectivity and estimating. Examples are direct rating or (interval) direct allocation, ranking (reverse, reciprocal, ...), (repeated) bisection, trade off, swing etc. For explanation is referred to the literature. The most thorough principle of comparison may be a complete systematic paired comparison (Pairwise Comparison Method, PCM) as promoted by Saaty and various co-workers in particular. Within PCM several scales have been proposed for scoring (e.g. Saaty, Lootsma, Salo and Hamalainen). Next to intuitive understanding and acceptability of this method, the possibility of a computable measure of consistency is an important advantage. The principle is to compare a matrix of weighting factors $[w_i/w_j]$ with a PC-matrix $[a_{ij}]$. The consistency is the deviation of the maximum eigenvalue of the rank of the matrix: reason supplements lacking empiric.

A noteworthy empirically often found aspect of value is that the value per extra added unit of quantity decreases and sometimes even approaches a maximum with increasing amount. This is attributed to the behaviour of people and is already recorded by Gossen in 1854⁵. Also the phenomenon of a minimal amount needed to obtain a value may occur. This is illustrated in figure 1, left the increase of awarded value y decreases with increasing quantity x and right lines of equal

³ Quality originally meant kind, later feature. And an adjective as 'poor' or 'high' is therefore necessary. The suggestion that quality is always high, is misplaced.

⁴ The Public Procurement Act (2012) provides a general legal framework for public procurement regulations and implements the relevant European public procurement directives 2004/18 Consolidated Public Sector Directive, 2004/17 Utilities Directive and 2007/66/EC Public Contracts Review Procedures Directive.

⁵ Hermann Heinrich Gossen (1810 – 1858) was an economist and worked until 1847 as an officer in the Prussian service. In 1854 he introduced the concept of diminishing marginal utility.

value of two options are shown. Both figures are related, as will be seen from the discussion of the linear normal model further on.

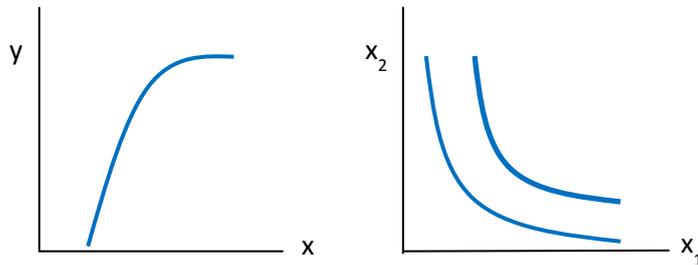


figure 1: empirical value functions

2.3 Price

Determination of a price does not occur in EMAT itself, but it does so in the preparation phase of an offer – where a seller should determine which quality and quantity versus which price he will offer – and in situations of tenders with negotiation or extension of a contract, as far as legitimated by the contract regulations.

In the general case a sale requires an agreement of two participants on a price. The reason to commit to a trade is that the value of a good differ for both participants. Profit is the difference between price and cost (producer, seller) and benefit the difference between value-for-the-buyer and price (buyer). In a balanced situation is (costs + profit) = price = (value – benefit). All five terms have the same unit, usual that of money, e.g. € or \$. A fair price for both participants would therefore be one between cost and value-for-the-buyer.

Confusion may occur because these terms are often used both per unit as for the total. It is recommended that in the first case to add 'per unit'. Another confusion can arise when the term 'price' is used for cost-for-the-buyer or value-for-the-seller.

The way in which a price emerges is shown in figure 2. The terms and symbols are

| | |
|----|------------------------------------------------------------|
| X | name of a good |
| x | size or sort of X, expressed in a number |
| y | value of X, e.g. the (maximum) possible yield or income |
| y' | unit value of X: $y' = dy/dx$ |
| k | cost, e.g. the (minimum) necessary acquisition expenditure |
| k' | unit cost of X: $k' = dk/dx$ |
| p | agreed price |

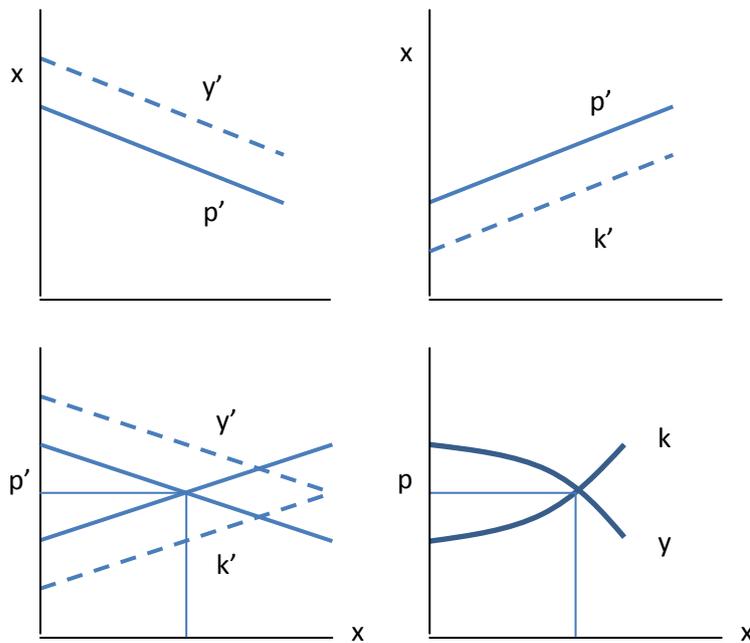


figure 2: price forming

The quantity of x that the buyer is willing to purchase, decreases with increasing unit price p' . But when the unit price rises, the seller on the other hand is willing to deliver more. By turning the axes and combining both a balance is found as the intersection of the price lines. At a sale where no profit or benefit and no loss is suffered by one of the participants, the amount x and unit price p' go to the intersection of the dashed lines. The total price p is determined next by integration of p' . The amount x does not change.

2.4 Choice

The word 'choice' is preferred in this context to 'decision'⁶. Though the final effect of both operations may be equal, their methods differ. Choice is the determination of an option with the highest value after assigning values to all the options. The core of the matter is a multi-criteria analysis (MCA). Several other names are in use, like by adding 'attribute' (A) and 'theory' (T) and replacing criteria by value (V) or utility (U), so MAVT or MAUT

Decision is successively removing options till one is left over. The name is then multi-criteria decision making (MCDM) with similar variations. A special case of (incomplete) decision is Pareto selection, which is a method of only rejecting options that score less than at least one other on all fronts. It may be that subsequently other methods must be used to come to a unique result.

For both methods many operational variants are developed. Examples of choice models are SMART and AHP/ANP and examples of decision models are ELECTRE and PROMETHEE.

Numbers may be classified as NOIR (nominal, ordinal, interval, ratio numbers). Cardinal numbers are of type I and R and the ordinal of course of type O. The purpose of a choice is to go from cardinal values (range) to ordinal (rank). The differences in cardinal scores between options is no longer of interest. An option is just ranked higher than another one if its ordinal score is higher. That can lead to surprising so called rank reversal problems, see the paragraph on rank reversal.

In figure 3 the choice problem is drawn for two options with variables x_1 and x_2 . The principle is to construct a value function $y(x_1, x_2)$, the blue plane. Lines with constant value y (red line in blue plane) represent options of equal value (red line in x_1 - x_2 plane).

⁶ The difference is also expressed in the original meaning of the words. Choice, a word from Germanic origin, is comparable with Latin lego, legere = collect, choose, read. Decision (Latin de-cadere = cut off, fall off) on the other hand comes to a result by subsequent rejecting options until one remains.

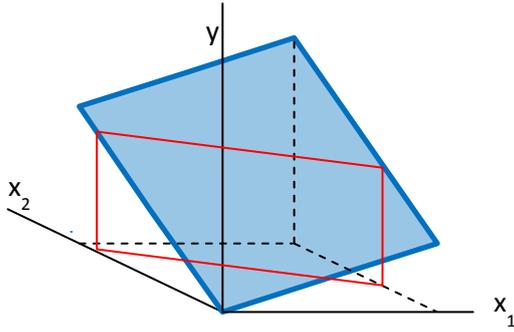


figure 3: schematised choice problem for two variables (In general the plane and the lines are curved)

3 Linear normal value function

Many value functions have been proposed. However, determination of the parameters of the simplest case of a bounded linear function of only two variables poses enough problems in practice to refer functions of more subtle and complex nature to the oddities. Also the rationale of more complex functions is usually questionable. Here, the value function is simplified on three points:

- The linear value function of a variable has 4 parameters, the x and y values of the beginning and end points or the replacement of 1 of these 4 by the slope of the line. This is the minimum information needed to describe one single attribute of an option. If a more complex functions would be known by empirical data, it is always possible to reduce such a function by linearization

$$y = \sum_i a_i x_i. \text{ (In case of a quotient is the Taylor series } \frac{1}{x} = \frac{x_0^{-1}}{0!} - \frac{x_0^{-2}}{1!}(x - x_0) + \dots = a_0 + a_1 x + \dots \text{).}$$

- Normalize all values on [0, 1]
- Positive gradients also make mathematics easier. If e.g. the sum of quality q (positively rewarded by the buyer) and price p (negatively rewarded by the buyer), so q - p is regarded, then $\frac{\partial u}{\partial p} < 0$, $\frac{\partial u}{\partial q} > 0$, and p may be replaced by the saving on the allowed budget $r = p_{budget} - p$, so $\frac{\partial u}{\partial r} > 0$

The three-step procedure to determine the best option is:

- determine the value functions v_{ij} for each attribute and thus variable x_{ij} , (2) and figure 4
- determine the choice model, i.e. the weight factors w_j , (3) and figure 5
- calculate the total (simultaneous) value function u_i of all attributes of option i, (1) and figure 5 and 6

The equations for this procedure are

$$u_i = \sum_j v_{ij} w_j \tag{1}$$

$$v_{ij} = \frac{x_{ij} - x_{ij, min}}{x_{ij, max} - x_{ij, min}} \tag{2}$$

$$w_j = \frac{y_{j, max} - y_{j, min}}{y_{max} - y_{min}} \tag{3}$$

$$u = \max(u_i) \tag{4}$$

with:

- i option
- j attribute of the option
- v normalized value function of the attribute per option
- w weight factor of the attribute
- u normalized simultaneous value function of the option

It is conditional that x_{ij} are independent. If $v_{ij} < 0$ the option is rejected. If $v_{ij} > 1$ the extra is usually allowed, but does not contribute to a higher value u_i . Due to (3) $\sum_j w_j = 1$, $w_j > 0$. Furthermore w_j also has to comply with the requirement of consistency. The preferred option is determined by (4). Formula (2) is shown in figure 4.

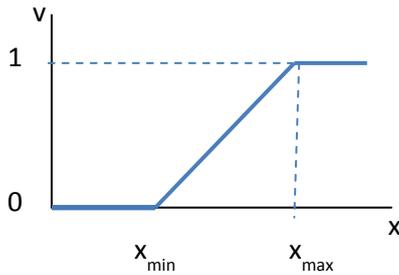


figure 4: linearized and normalized value function v

The linearized normalized value function for 2 variables is shown in figure 5. The weight functions are the coefficients of directions (i.e. the tangents of the slope angles).

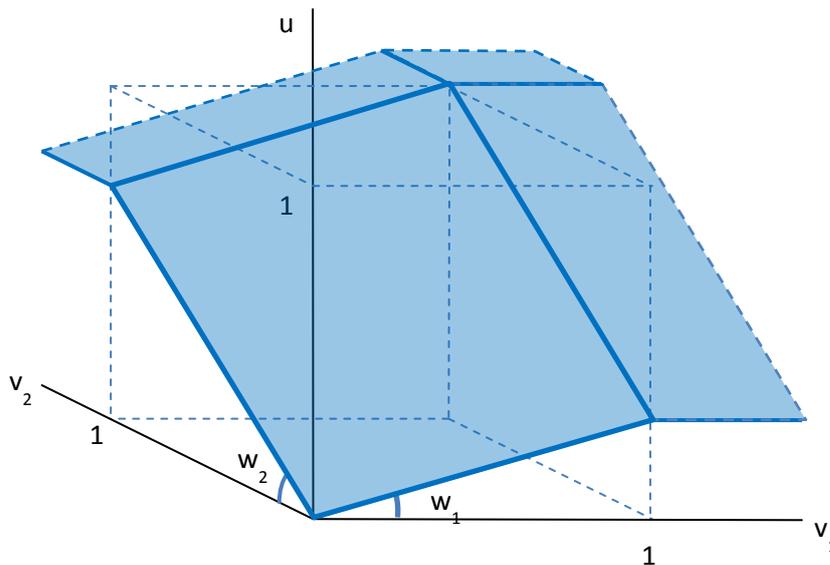


figure 5: linearized normalized value function

The choice problem is the projection of equal-value lines on the ground-plane. The higher lines are more preferable.

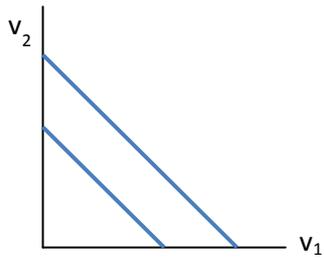


figure 6: projection of lines of equal y for combinations of v

In case of a non-linear model like Gossen, a partial-linear equation may be used. Only the sloped part is relevant for EMAT.

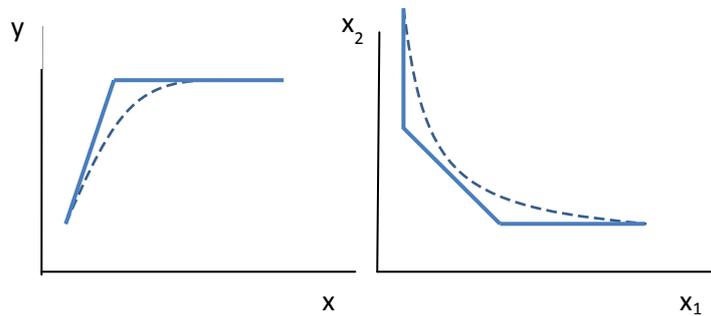


figure 7: projection on a plane of constant utility

In table 2 a simple example of 3 criteria X_j and 3 offers X_i is shown. The numbers are fictitious. The client (buyer) has to complete the yellow cells and the information in the blue cells has to be delivered by the candidate contractors (seller).

table 2: example

| aspects | unit | | x_{i1} | x_{i2} | x_{i3} | w |
|--------------------|---------|----------|----------|----------|----------|------|
| offered price | 150 k€ | x_{i1} | 1 | 3 | 7 | 0,65 |
| duration work | 20 days | x_{i2} | 0,33 | 1 | 5 | 0,28 |
| designed life time | 1 year | x_{i3} | 0,14 | 0,2 | 1 | 0,07 |
| | | | | | ew | 3,06 |

| required | | | | | offered | | |
|----------|------|-------|------|------|----------|----------|----------|
| aspect | unit | worst | best | w | x_{1j} | x_{2j} | x_{3j} |
| x_{i1} | k€ | 1500 | 1000 | 0,65 | 1050 | 1375 | 1450 |
| x_{i2} | dag | 200 | 100 | 0,28 | 110 | 175 | 190 |
| x_{i3} | jaar | 5 | 11 | 0,07 | 10,4 | 6,5 | 5,6 |

| aspect | normalisation | | | weight | score | | |
|----------|---------------|----------|----------|--------|-------|-------|-------|
| | v_{1j} | v_{2j} | v_{3j} | | u_1 | u_2 | u_3 |
| x_{i1} | 0,90 | 0,25 | 0,10 | 0,65 | 0,58 | 0,16 | 0,06 |
| x_{i2} | 0,90 | 0,25 | 0,10 | 0,28 | 0,25 | 0,07 | 0,03 |
| x_{i3} | 0,90 | 0,25 | 0,10 | 0,07 | 0,06 | 0,02 | 0,01 |
| sum | | | | 1,00 | 0,90 | 0,25 | 0,10 |

4 Consequences

4.1 Other models

There are many equations in circulation which are used as models for EMAT decisions. (7) – (11) are five examples of equations with two independent variables, p and q, e.g. representing price and a particular quality. The main differences between these equations are the number of parameters – i.e. the amount of knowledge needed for the decision-making problem - and the difficulty of obtaining data. In all models, assumptions for the domain of the variables are often kept implicit (or not at all) and thereby not accessible for judgement.

For comparison, the normal linear model for the two variables is expressed in terms of p and q too (5). For this case and for the approximations of the Gossen cases, the price is converted to the remaining budget after paying the price r, (6).

$$u = w_r v_r + w_q v_q \quad (5)$$

$$r = p_{budget} - p \quad (6)$$

$$U = p_e = -p + cq = p_{budget} + r + cq \quad (7)$$

$$U = \frac{q}{p} \quad (8)$$

$$U = r^m + cq^n \quad \text{or} \quad U = r^m q^n \quad (9a, b)$$

$$U = \left(1 - \frac{\lg\left(\frac{p}{p_{min}}\right)}{\lg\left(\frac{p_{max}}{p_{min}}\right)} \right) + cq \quad (10)$$

$$U = \left(0,5 \left(\left(\frac{p}{p_r} \right)^n + \left(2 - \frac{q}{q_r} \right)^n \right) \right)^{1/n} \quad (11)$$

The preferred option in (9) is determined by maximization and in (10) by minimization. The symbols are:

- p price offered
- p_e equivalent price (often indicated as a fictional offered price)
- p_{min} lowest price offered (highest in case of index 'max')
- q quantity
- U value of the offer
- c coefficient (weighting factor) relative p-q
- r (index) reference
- n (exponent) value to be chosen, eg. 3 to 5

For (7), (8) and (9) a visual impression is seen in the figure below.

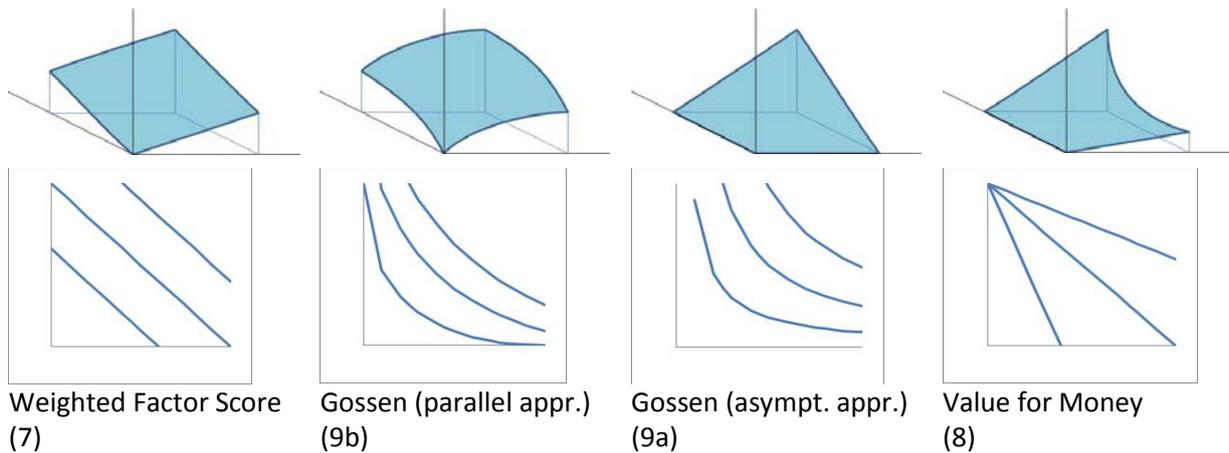


figure 8: comparison of some models (top: value plane, bottom: lines of equal value)

(7) is a so called Weighted Factor Score (WFS). Problems are the determination of c and the limits for p and q . Often, much attention is paid to the lower limit for q , usually in a Specification of Requirements, while also an upper limit for p and, to a lesser extent, for q are desirable. Also understanding of a lower limit for p will also be useful in the assessment. Too low prices mean shifting problems to later project phases in practice. This model is already long-time known as Multi-criteria analysis (MCA), Cost Benefit Analysis (BCA) etc.

(8) is called Value for Money (VFM). This corresponds to the definition of Value Engineering: value = functionality/cost, already suggested in ca. 1944 by Miles. Though often overseen, parameters still need to be determined to capture limits for p and q . Excessive p and q encounter problems that are not included in this model, eg. a budget. A fundamental problem is that all lines of equal value pass through one point. There is no reason in reality for that. Furthermore, the unit can be confusing when more than one quality is used.

(9) are possible formulations of the Gossen model, only for reason of comparison to the other equations. Data is usually found too inconsistent to generalize to equations.

(10) is suggested to avoid the problem of the so called rank reversal. 4 parameters are to be chosen. The meaning of the lg-transformation is that $lg(p/p_r) = lg(p) - lg(p_r)$, with which a price difference becomes independent of the choice of a reference price.

(11) is proposed by the Netherlands tax authorities. The derivation of the form and the arguments for the factors is unclear. (11) is a principle already known since 1961 (Arrow c.s.) as CES (Constant Elasticity of Substitution) production function $Q = (cK^n + (1 - c)L^n)^{1/n}$ with e.g. Q production, K capital and L labor. The argument of CES is that the elasticities, the relative derivatives, retain the form $\frac{dQ}{Q} = \left(cK^n \frac{dK}{K} + (1 - c)L^n \frac{dL}{L} \right)^{1/n}$.

4.2 Rank reversal

A properly made choice is replacing a rate by a rank (while an improper choice is just a guess). In the theory of decision making and in legislation and jurisprudence, much attention has been given to the phenomenon of rank reversal⁷, i.e. the ranking of top-candidates is effected by low ranked candidates. The best known example is that withdrawal of the lowest placed candidate can change the ranking of the others⁸.

⁷ The sometimes used word 'rank-order-paradox' seems a bit awkward. First, it is sufficient to use in the term either rank (of Germanic origin) or order (Latin). Second, there is no paradox, but a possibly somewhat surprising result of a calculation. There exists a decision-paradox, namely that to make a decision first the decision-method to be used has to be chosen. The outcome of a decision may depend on the method chosen next to or even instead of on the attributes of the options.

⁸ Wikipedia mentions 5 types of rank reversal, of which 1 and 2 seem identical and related to the case described here. 3 and 4 again seem identical and concern the degree of detail of the hierarchical form of the

This phenomenon is for the linear normal model made clear in figure 9. In the right figure a change of the lower limit (e.g. withdrawal of the lowest placed candidate) shifts the value function from the dashed to the drawn line. In the v_1 - v_2 plane, with lines of constant value, A changes from the higher rated dashed line to the lower rated drawn line and B does vice versa. As a result their rankings change.

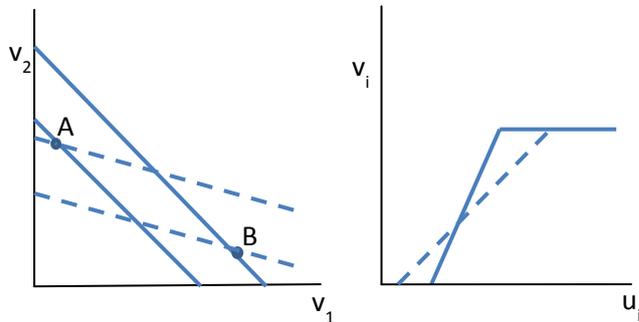


figure 9: example of grade change as the lower limit is changed

4.3 Dependence and sensitivity

A major problem in all models may be dependence between variables. Due to overlap in meaning multiple-counting may occur. Such overlaps are sometimes (often) hard to discover and even when discovered, hard to prevent to effect the outcome. An often seen example is volume discount, where quantity and price are related. An explaining mechanism is that fixed costs are distributed over unit costs and thereby lower the unit price.

Sensitivity or robustness means that changes in the input have not unrealistic small or big effects on the outcome. Account must also be taken of the discrimination of the model to avoid pure formal differences, in which case even the reprehensible lottery might be fairer, on the one hand and overseeing meaningful differences on the other. A difference has to be meaningful to avoid two options being ranked in an unjustified manner. Extreme value-, scenario- and probabilistic analyses are methods to investigate sensitivity and discriminative abilities of the model.

5 Conclusions

EMAT is basically well-known classical microeconomics, which should be known by persons who implement an EMAT-assessment and kept in mind during the process. Much current discussion can be reduced by first taking notice of this long existing knowledge (this is a risky note that may easily turn against the author).

Simple EMAT score functions are recommended. More complex models often can not be underpinned, sufficiently substantiated and explained in practice. Data can often not be obtained economically or is lacking or insufficient at all. A (piecewise) linear function is hard enough in practice.

The buyer must determine both values v weighting factors w in an insightful manner. That is not an easy task and moreover has subjective elements. This requires expertise and responsibility. Buyers should be hold responsible in case of defects.

The purpose of EMAT is to rank options. That is often done by giving values to these options (rating). Rank reversal can occur in at least three ways: interdependence of v and w , combinations of relative scores and change to another model (the so-called decision-paradox).

considered decision problem. 5 is evident: when using a different model, the results turn out different (this is paradoxical).

Dependency between attributes is common and can have great effects, but is rarely acknowledged. Over- or underestimating sensitivity may have deleterious outcomes. Analysis and trial calculations are recommended.

Practical assessments may evoke the impression of arbitrariness. It is the obligation of the buyer to prevent or, if occurring, to vitiate such suspicions, because he has chosen the tender method.

Buyers must inevitably make choices, which requires expertise and responsibility and for which they should be held responsible.

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